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NEW ART of SURVEYING

B Y T H E

GONIOMETER;

CONTAINING,

- I. A New METHOD of measuring Angles by this New Instrument with much greater Exactness than can be done by a PLAIN-TABLE or THEODOLITE; which is at the same Time more Simple, Portable, and less Expensive.
- II. The DESCRIPTION and Use of a *New* PROTRACTOR, Double PARALLEL-RULER, PANTAGRAPH, and other Instruments used in *Plotting off*, and *Copying* SURVEYS in any Size.
- III. The Rationale of reducing any *multangular Figure* to a *plain Triangle*, and thereby finding the CONTENTS of a SURVEY by one single Computation.
- IV. The PRINCIPLES of LEVELLING in THEORY and PRACTICE concisely explained.

The Whole illustrated by a great Variety of
COPPER-PLATE FIGURES.

By BENJ. MARTIN.

L O N D O N:

Printed and Sold by the AUTHOR in *Fleet-Street*.

M DCC LXVI.

Benj. Martin

T H E

New Art of SURVEYING

B Y T H E

GONIOMETER;

CONTAINING

- I. A New Method of measuring Angles by this New Instrument with much greater Exactness than can be done by a Plain Table or Trigonometrical; which is at the same Time more Simple, Portable, and less Expensive.
- II. The Description and Use of a New Reflecting Circular Parallel Rule, Pantagraph, and other Instruments used in leveling off, and Geometrical Surveys in any Style.
- III. The Methods of reducing any unknown Figure to a plain Square, and thereby finding the Contents of a Survey by one single Computation.
- IV. The Principles of Levelling in Towers and Places exactly explained.

The Whole illustrated by a great Variety of Copper-Plate Figures.

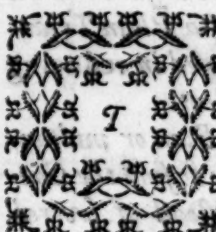
By JOHN MARRAS

L O N D O N

Printed and Sold by the Author in Fleet Street.
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P R E F A C E.

HE Subject of LAND-SURVEYING has been long since exhausted with Respect to the common Forms, both of PRACTICE and INSTRUMENTS. But as in both these Cases there have been very considerable Improvements made, I judged an Essay to explain the Nature and Application of them, would be agreeable to, and gratefully received by the Public.

The common Method of Surveying by the best of Instruments, viz. the THEODOLITE, it is well known, will not give an Angle to greater Exactness than of about three Minutes, in a Radius of 5 Inches, which is the usual Size. The Price of which is Twenty Guineas in SISSON's Form. Now the Instrument here described and recommended, is that improved by the late learned Mr. Mayer of Gottingen, under the Title of a GONIOMETER, which may be made of a cheap and portable Form, and yet shew the Angle to the Accuracy of 15 or 20 Seconds, which is 10 or 12 times nearer the Truth than by the Theodolite.

Besides a little more Expence will give a new, and very considerable Improvement of Mayer's GONIOMETER; and then it will be little more than half the Expence of SISSON's Theodolite. The Artifice by which this Degree of Accuracy is attainable by this Instrument, we have largely explained in the following Tract, and illustrated the Process in all its Particulars, by an entire Copper Plate on the Subject.

As the Precision in taking Angles is the Essential Part of the Art, so the greatest Care ought to be taken about it, as the Truth of the SURVEY entirely depends upon it. This is therefore made the Capital Subject of the ensuing Treatise.

Some other Instruments of less Moment, have yet required our Notice, as they are either capable of a better Form, or require



require to have their Theory explained from proper Principles. The Double PARALLEL RULER with its TABLE, it is hoped will be found in Practice a very great Improvement. As also the New PROTRACTOR for laying off Angles to a Minute in plotting the Survey, with the genuine Theory of that Invention.

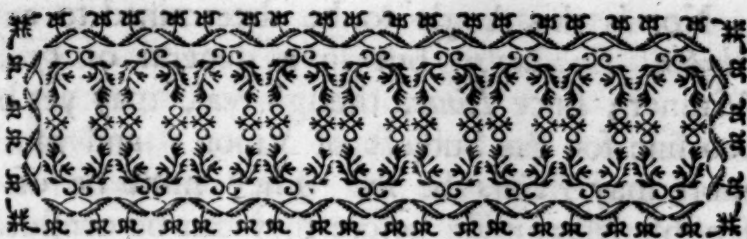
We have likewise added the Theory of three different Sorts of PANTAGRAPHS from the Properties of a Plain Triangle only; by which the Reason of the Process used in reducing and enlarging Plans will be clear and evident.

The Method of reducing any Polygonal Plan or multangular Figure to one plain Triangle, in order to find the Contents of the Survey by one Operation instead of several, we thought of too much Consequence to be passed over without some farther Ecclaircissement than what is generally met with in Books on this Subject.

As LEVELLING is generally considered a Part of the Province of a SURVEYOR, so we have here given it a Place, together with its THEORY, as Hammond, Wild, and others, have been too silent about it. A Person who is not acquainted with the Rationale of his Art must always be considered as working in the Dark. The SCIENCE of any Profession consists in the Theory, and they who are content to be ignorant of that, have mean Souls, and are not to expect the Honours attendant on MEN of SCIENCE.

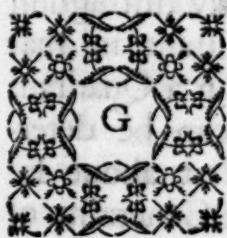
In order to make every Thing easy to the Access of every ingenious Artist, we have made it a Point to make the Expence and Bulk of the Book small; by leaving out all the Minutiae of this Art on one Hand, and all the Redundancies on the other; it would be affronting to suppose any Surveyor wanted to be taught the first Principles of Geometry; and little less than an Imposition to oblige him to pay for large Copper-Plates for taking the Perspective of a Gentleman's House only, which is very foreign to the Use of the Theodolite, and can only be done well by GLASSES, or by a Perspective TABLE.

To conclude, I am persuaded that every Part of this Essay wears so much the Face of Novelty, that it may justly be thought to deserve the Title of the New ART of SURVEYING.



C H A P. I.

The PRINCIPLES of GONIOMETRY defined and explained.



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ONIOMETRY is the Science of estimating or measuring the *Quantity of Angles*, subtended by distant Bodies, or Arches of Circles, by Instruments constructed in the best Manner according to the Rules of Art.

The *Quantity* of an *Angle* is the *Arch* of a *Circle* which is intercepted by two right Lines drawn from the extreme Points of any Object to the Eye of the Observer; for such an Arch of a Circle, be the Radius great or small, will be the true Measure of the Inclination of those Lines, to each other, which make the Angle under which the Object appears, which is therefore called the *optic* or *visual* Angle.

To measure or estimate this Angle to a *geometrical Exactness*, is impossible; as *geometrical Lines* and *Points* are Subjects only of our *Intellects*, and not of our *Senses*. We can image them to the Mind, but can by no Means represent them to the Eye.

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Nor is this Angle to be determined to any *physical Exactness*, but in a general or gross Manner; since *Lines*, though real, may yet be too fine for the Subject of Vision; and much more may *Points* be so. They must be of a sensible Magnitude before they can be considered, and used, in such Determinations as mathematical Instruments are concerned in.

And after all, the most keen and perfect Eye would be able to do but little of itself in determining the Quantity of an Angle; for a Degree of a Circle twelve Inches diameter is but about the *Tenth of an Inch* wide, and it must be a very good Eye that can clearly distinguish ten Points or Lines drawn equidistant in that Space. Whence it appears, that in a Quadrant of six Inches Radius, an Angle can be determined to no greater Accuracy than about six Minutes of a Degree; and, by the Generality of Eyes, to not less than ten Minutes. This is too gross an Estimation to be of any considerable Use.

If the Quadrant be twelve Inches Radius, even then five Minutes will be too far distant from Truth to render such an Instrument of any more than common Use. When they much exceed this size, they are neither *manual*, nor *portable*; and the Expence and Trouble of large Quadrants without artificial Assistances and Contrivances, will not be compensated by any Use they can afford, and are therefore now quite laid aside.

But many and great have been the Invention of Artists in this Age, to render Quadrants, even of a small Size, of extreme Use in the measuring an Angle to great Exactness. In-
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so much that very large Quadrants are now considered as very little necessary, as will appear in the Sequel of this Discourse.

The first great Reduction of the Size of the Quadrant, was that by Reflection from Glasses, invented first by Sir ISAAC NEWTON, and afterwards improved into the modern Form of the *Sea-Octant*, usually called *Hadley's Quadrant*. This *Catoptric Quadrant* measures an Angle with half the usual Extent of Limb; and is moreover fitted for many uses, particularly at Sea, by a Property which no other Instrument can have, which is largely explained in my *THEORY of HADLEY'S QUADRANT*, to which the Reader is at present referred.

The *DIAGONAL DIVISION* of a DEGREE on the Limb of a Quadrant is an excellent Contrivance, and will enable the Artist to estimate an Angle to a *Minute of a Degree*, nearly, on the Limb of a twelve Inch Radius; and to two Minutes on that of the *Sea-Octant*. The Nature of these *Diagonal Lines* being in general well understood, I need not stay here to describe them.

This Improvement in GONIOMETRY was succeeded by another still greater, which is that Division of an Arch by what is commonly called a *Nonius*. But this Invention is really owing to one *VERNIER* a *Spaniard*; and is now used and applied to all Quadrants and Octants where great Accuracy is required in measuring Angles, as in most Cases of SURVEYING, NAVIGATION, and ASTRONOMY. For by the *Nonius* you measure an Angle with twice the Exactness, and much more Ease, than by *Diagonals*. The Nature and Use of the *Nonius*, or *Vernier's*

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Divisions are also explained in the before-mentioned *Traët on Hadley's Quadrant*.

Though I have never seen an Instrument in which both these Methods of dividing a Degree, or rather of measuring an Angle, were employed at the same Time, and so as to multiply each other's Effects, yet I was resolved to attempt such a Construction; and by the Event I find a Minute may be divided into *Seconds* in the same Manner as a *Degree* is divided into *Minutes*. And of Course an Instrument which has a Semi-circle only of five Inches radius, and fourteen Degrees of the Limb of another Circle twenty Inches radius, will, by Means of the above Divisions combined, measure an Angle to a Second; at least this would be the Case if our Eye-sight were but as perfect as the Instrument itself.

But such is the Ingenuity of modern Artists, that they have so far improved artificial Constructions, as even to exceed the Power of Nature, without Improvement also, to follow or apply them. For it is certain the Angle that is measured by the *Goniometer* must, at the same Time, have its Limits (*viz. where to begin and end*) determined by the Eye, either *alone*, or *assisted* also by *Art*. The Eye alone does this by *plain Sights*, as they are called; and the perpendicular Hair in the *Sight-vane* is made to lie upon, or coincide with, some particular Spot or Line in one Object, for the *Beginning* of the Angle; and in another Object (or in a different Part of the same) the like Coincidence must be observed for *finishing* the Angle.

Hence it appears that the Truth of the Angle measured by the Instrument, depends on the
Exactness

Exactness of the observed Coincidences of the Line in the *Sight*, and of the Point or Line in the Object. Therefore we in vain talk of great Improvements in the accurate Construction of Instruments, after they exceed the natural *Poignancy* of *Sight*, unless this also can be proportionably improved at the same Time.

But it is here to be observed, that when I speak of a Point, Spot or Line in the Object, I do not mean a *shining or lucid Point*, as that of a *Star*, which is easily seen without subtending any Angle at all; but a visible Spot on any common Object, which is not sensible unless it subtends an Angle of nearly *one Minute*; at least, of *forty Seconds*. And by a *LINE* I mean not a *single Line* placed in the most advantageous View which may be visible, though it subtends an Angle of not more than three or four Seconds, as Dr. JURIN has observed; but I mean such Lines on the Surfaces of distant Objects as we can just discern with the naked Eye sufficiently for determining the Coincidence of the Line in the *Sight* therewith.

And because these Points and Lines must at least subtend an Angle of *one Minute* to be themselves distinctly visible, it follows that by *plain Sights* we are not to expect to be nearer the Truth than *one Minute* at *either Limit* of the Angle; and consequently, the true Quantity of the Angle cannot be approximated to a greater Accuracy, than that of *two Minutes* by the Eye alone; and therefore not by the Instrument in the general Cases of *Surveying*, be its Construction what it will.

The late celebrated Astronomer Mr. MAYER who has wrote professedly on the Structure

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ture and Excellency of the GONIOMETER, has supposed that Lines and Spaces equal in Breadth, and equidistant (as in Fig. 8. of Plate I.) cannot be distinctly viewed at a less Distance than thirty Inches, though each Line or Space be $\frac{2}{10}$ of a *French* Line (or $\frac{1}{3}$ of an Inch) in Width, and consequently at that Distance the Angle subtended by it will be $1' 54''$.

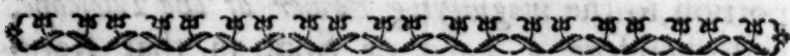
But there is a wide Difference between viewing a single Line distinctly; and many placed near to each other, or the Spaces between them; for the Line by itself will appear sufficiently evident for a sensible Coincidence with another, at ten times the Distance that the equal Space between two such Lines can be distinctly seen. So that we still suppose the Eye may estimate an Angle, or ascertain its Limits, within two Minutes of a Degree; and this with regard to common opaque Objects; but with respect to the heavenly Bodies, particularly the *Stars*, the Eye alone by *plain Sights* is capable of determining the Angle almost, if not quite, as exactly as when assisted by Glasses. Dr. HOOKE therefore, with very little Reason clamoured so loudly against the Method of *plain Sights* used by the noble HEVELIUS in his Observations of the *Stars*.

But with regard to *Geodesia*, or LAND SURVEYING, and all Kinds of LONGIMETRY, the natural Eye-sight ought to be assisted as much as possible, to determine the Limits of the Angle to be measured; and this is done by a common TELESCOPE extremely well. For the Line in the *plain Sight* is now supplied by the fine Hair or Wire in the Focus of the Eye-glass, and the *Spot* or *Line* in the Object to coincide

cide therewith, will now be enlarged in Proportion to the *magnifying Power of the Telescope*, and therefore a Line or Spot in that Ratio less will suffice for a Coincidence, and consequently for fixing the Terms, or Quantity of the Angle.

Thus for Example, suppose the Telescope magnifies an Object ten Times, then a Line or Spot ten Times less in Diameter than before, will now be equally visible, and the Coincidence equally evident, and therefore ten Times nearer the Truth; consequently each Limit of the Angle will now be within six Seconds of the Truth, and the whole Angle determinable to within twelve Seconds; and in proportion as the Telescope magnifies more or less, so will the Accuracy be of determining the Angle by the Eye thus assisted. And this Error of *Collimation* (as 'tis called) as it may equally happen in *Defect* or *Excess*, may be looked upon as nothing at all, or so far diminished in the several Operations, as to be not worth considering.





C H A P. II.

*The ARTIFICE explained, upon which
the USE and PERFECTION of the
GONIOMETER depend.*

 E are next to see that the Instrument keep Pace with the Eye in its Improvement for measuring Angles ; and here lies all the Difficulty. To do this by a *single Nonius* would require a very large Radius ; if the *Nonius* be combined with *Diagonals*, the Instrument will be *complexed*. A *Nonius* and a *MICROMETER* will both inhanse the Price, and make an Instrument too large for a *portable Form*. In short *mechanical Contrivances* will very little avail us in an Enquiry of this Sort. Artifices of another kind must, therefore, be sought after ; and happily for the *Surveyor*, there is one Resourse from whence he may derive almost what Degree of Accuracy he pleases in all the Angles he measures.

The Artifice hinted at, is this, viz. by *measuring the Angle several Times over, and then taking a Mean of them all*. This Method is found by Experience to be extremely accurate, and to render large and expensive Instruments altogether unnecessary ; Mr. MAYER has given the Form of the GONIOMETER compendious enough, with a SCALE *Diagonally* divided to every

every two Minutes of a Degree ; and then by the Multiplication of the Angle to be measured, it finds the true Quantity thereof to within 15" or 20" ; which is a Degree of Precision never yet known, or scarcely to be expected in a *Goniometrical Instrument* of no more than *six Inches Radius*.

The Use of this Instrument we shall describe and illustrate by a Copper Plate, having first premised that the THEORY thereof is derived from the DOCTRINE of CHANCES, and has been compleatly delivered by the late Mr. SIMPSON.* In this Theory it is supposed that the Errors of the Instrument are equally liable to happen in *Excess*, and in *Defect* ; and that they may be contained between certain assignable Limits, which depend on the Goodness of the Instrument, and the Skill of the Artist or Observer.

He then supposes the Instrument so large, and perfect, that every Observation may be relied upon to *five Seconds* ; and then enquires what the Probability or Chance for an Error of 1, 2, 3, 4, or 5 Seconds will be, when, instead of relying on *one*, the *Mean of six Observations* is taken ? The Answer resulting from his Theory, is a Ratio of $2\frac{1}{2}$ to 1 ; that, is the *Probability that the Error* by taking the Mean of six Observations, does not exceed a *single Second*, is measured by the Fraction $\frac{788814800}{1088351168}$, and therefore the Odds will be as 788814800 to 299576368 or nearly as 2,5 to 1. But the Odds when *one* single Observation is taken, is only as 16 to 20, or 8 to 10, that is $\frac{8}{10}$ to 1. But $0,8 : 2,5 :: 1 : 3,125$. So that you are *three times*

* See his *Miscellaneous TRACTS*, Pag. 64.

times less liable to an Error of one Second by a Mean of six, than by a single Observation.

Then the Enquiry is made for the Odds, or Proportion of the Chances, that the Result by a *Mean of six Observations* comes within *two Seconds* of the Truth; and it is found to be as 29 to 1 nearly; whereas the Odds by a *single Observation* is only that of 2 to 1; so that the Chance for an Error of 2'' in the first Case is not $\frac{1}{11}$ Part so great as in the latter. And in the same Manner it is found that the Chance for an Error of 3'' is not $\frac{1}{1000}$ (one thousandth) Part so great by a *Mean of six*, as by a *single Measurement* of the Angle.

Whence our learned Author concludes, "Upon the Whole of which it appears, that, the taking a Mean of a Number of Observations greatly diminishes the Chances for all the smaller Errors, and cuts off almost all possibility of any large ones: Which last Consideration is alone sufficient to recommend the Use of the Method, not only to *Astronomers*, but to *all others* concerned in making Experiments or Observations of any Kind, which will allow of being repeated under the same Circumstances."

What has been said of an Error of five *Seconds* holds equally for an Error of five *Minutes*, or five *Half Minutes*, *Quarter of Minutes*, &c. and therefore is applicable to Instruments of all Sizes. We shall first give a Description of the Method of taking the Multiple of an Angle by a GONIOMETER of that Form which Mr. MAYER has described and recommended.

This Instrument consists of two Brass Rulers *a b* and *c d* (*Fig. 1.*) of which the first *a b* is fastened to a Socket, moveable in a horizontal Position

Position upon a three legged Staff like a common Theodolite; and to be fixed in any Position required. The Ruler $c d$ moves about the Center upon the other, and carries a Telescope for Observation; on the Ends of each Ruler are five Holes made at equal Distances from the Center O ; as at a , b , and at c , d .

When these Rules or Bars are opened to any Angle, as $a O c$, or $b O d$, that Angle is measured by taking the Distance of the Points b , d , with a Pair of Compasses, and applying the same to a SCALE properly constructed for that Purpose, and by the *Diagonal Lines* of this Scale, the Quantity of the said Angle will be given to about *two Minutes* of a Degree; which Error of two Minutes is yet *much less than the common Theodolites produce.*

Now this Error by one Observation will be diminished by being divided among many; in $2'$ are $120''$, one sixth Part of which is $20''$, therefore by a Mean of six Observations or Measurements, you are three times less liable to an Error of $20''$, than by one single Measure. And the Odds against an Error of $40''$ are 14 to 1 . And against an Error of $60''$, or 1 Minute, as 1000 to 1 , as we have before shewn.

So that by taking a Multiple of the Angle, you diminish the Error of the Instrument so far as to render it inconsiderable, and, at the same Time, are very secure against committing any Error considerable enough to do Harm. And how properly this GONIOMETER is constructed to answer this Purpose of multiplying an Angle without Trouble, Hazard, or Loss of Time, will appear from the following De-

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tail of the Process, and proper Representation of each Step of the Operation in the Copper Plate.

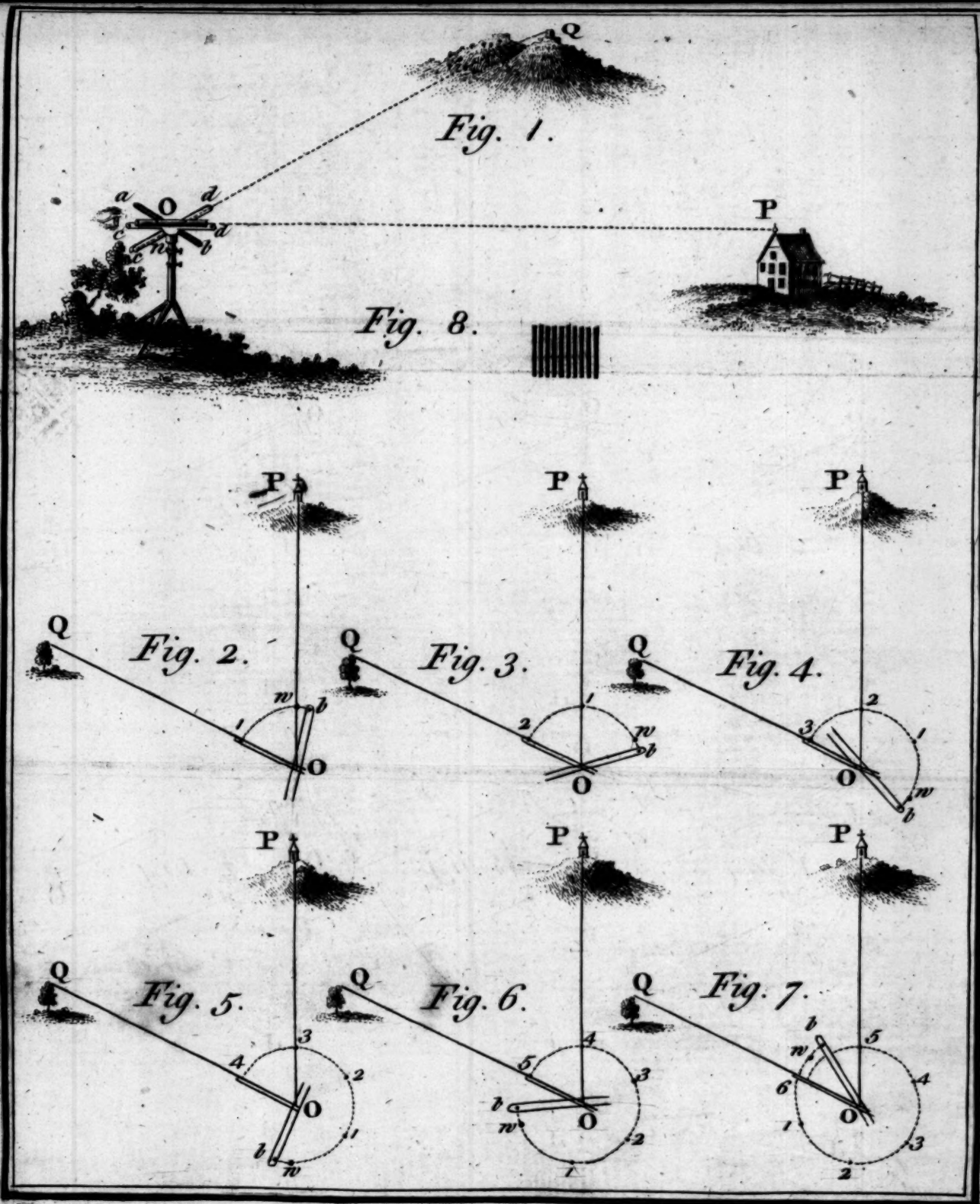


C H A P. III.

The Practical Use of the GONIOMETER explained and exemplified.

IN order to explain and exemplify the Use of the GONIOMETER in this new Way of Surveying, let P, Q be two Objects (*Fig. 1.*) whose Angular Distance is to be determined at the given Point O. The Goniometer being rightly fixed at O, the lower Ruler (*a b*) is to be so placed as to fall a little without the Limits of the Angle, and is there made fast by a Screw; then the upper Ruler (*c d*) with its Telescope is to be directed to the Object P; and when it is nicely upon the vertical Line in the Focus of the Glais, it is there fixed; and the Angle $a O c = b O d (= b O P)$ will be found by applying the Distance of the Points *a, c*, or *b, d*, (for they ought to be equal) to the diagonal Scale, and thus you have what may be called the *auxiliary Angle b O d*.

After this (the Ruler *a b* still remaining fixed) the Ruler *c d* is directed to the Object Q till it coincides with the vertical Wire of the Telescope; then the Angle *b O d* will in this Case also



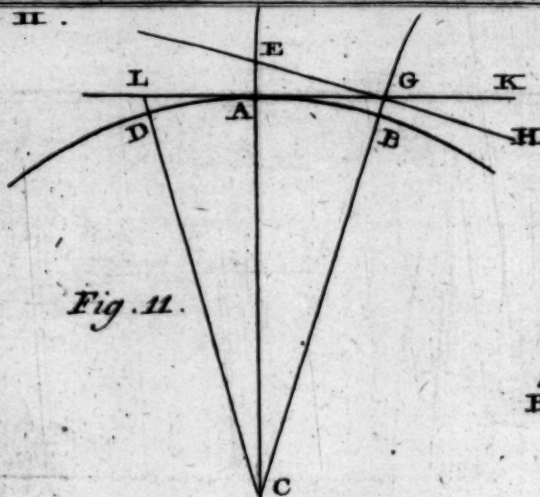


Fig. 11.

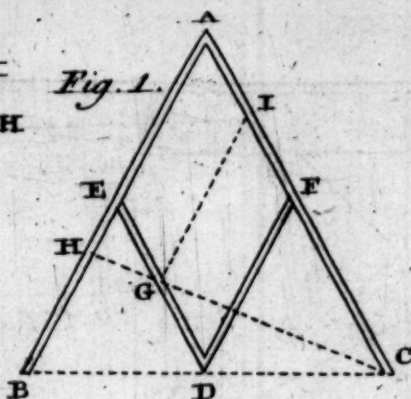


Fig. 1.

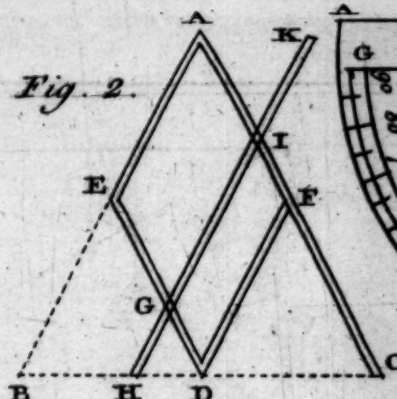


Fig. 2.

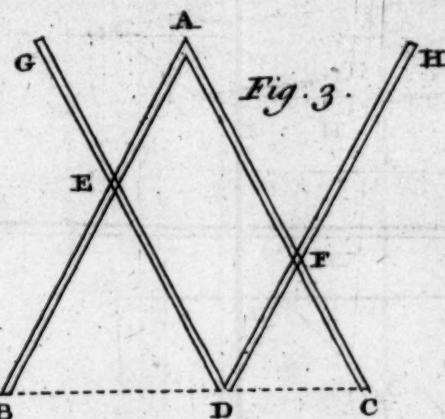


Fig. 3.

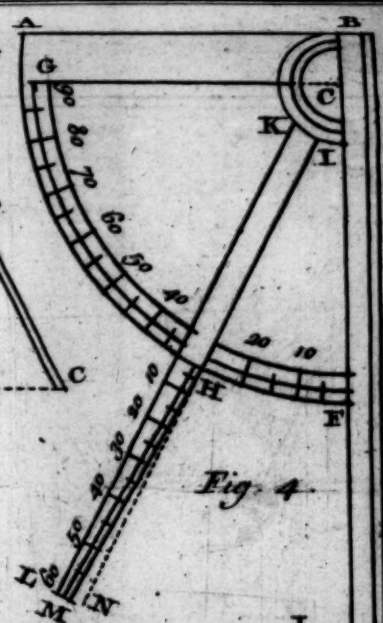


Fig. 4.

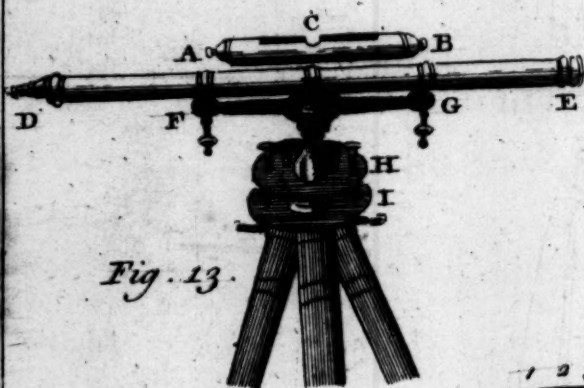


Fig. 13.

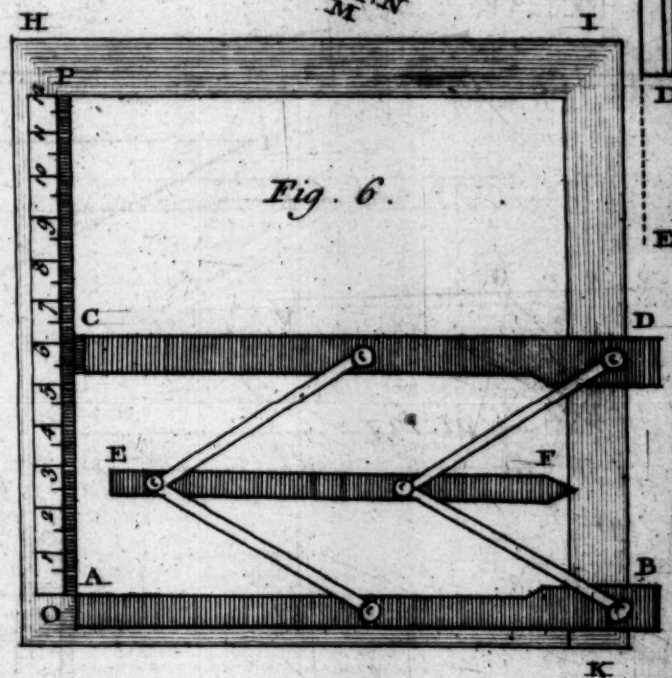


Fig. 6.

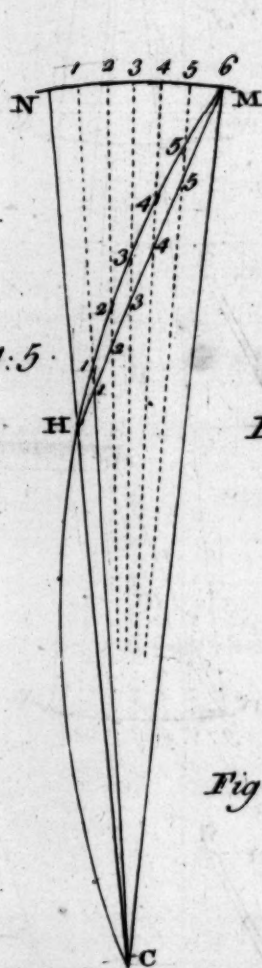


Fig. 5.

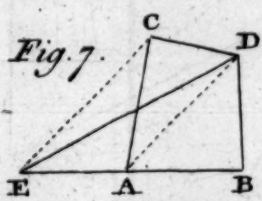


Fig. 7.

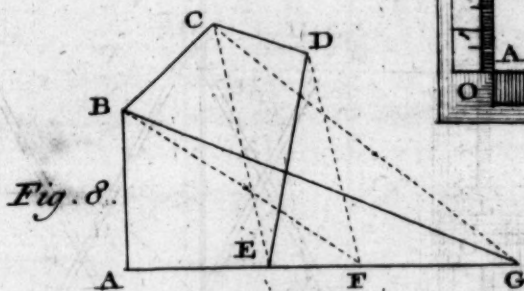


Fig. 8.

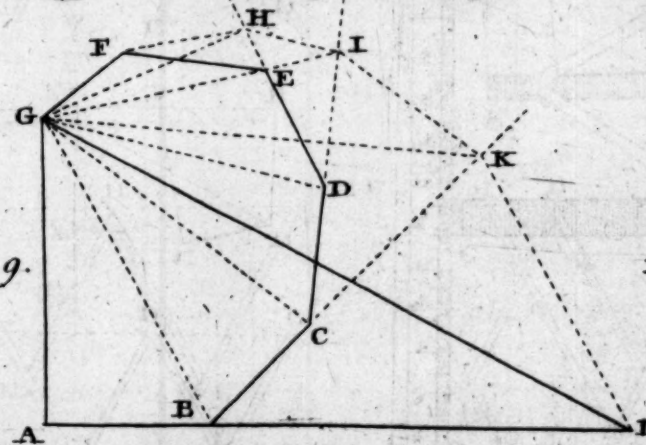


Fig. 9.

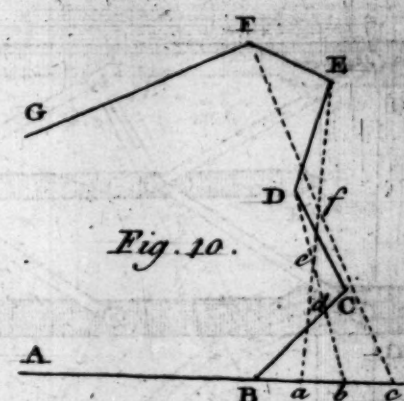


Fig. 10.

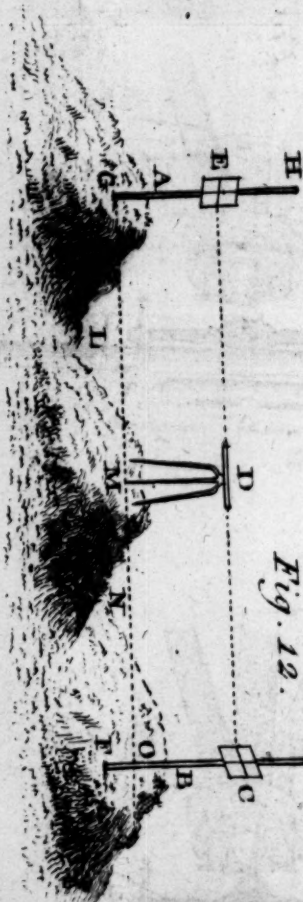
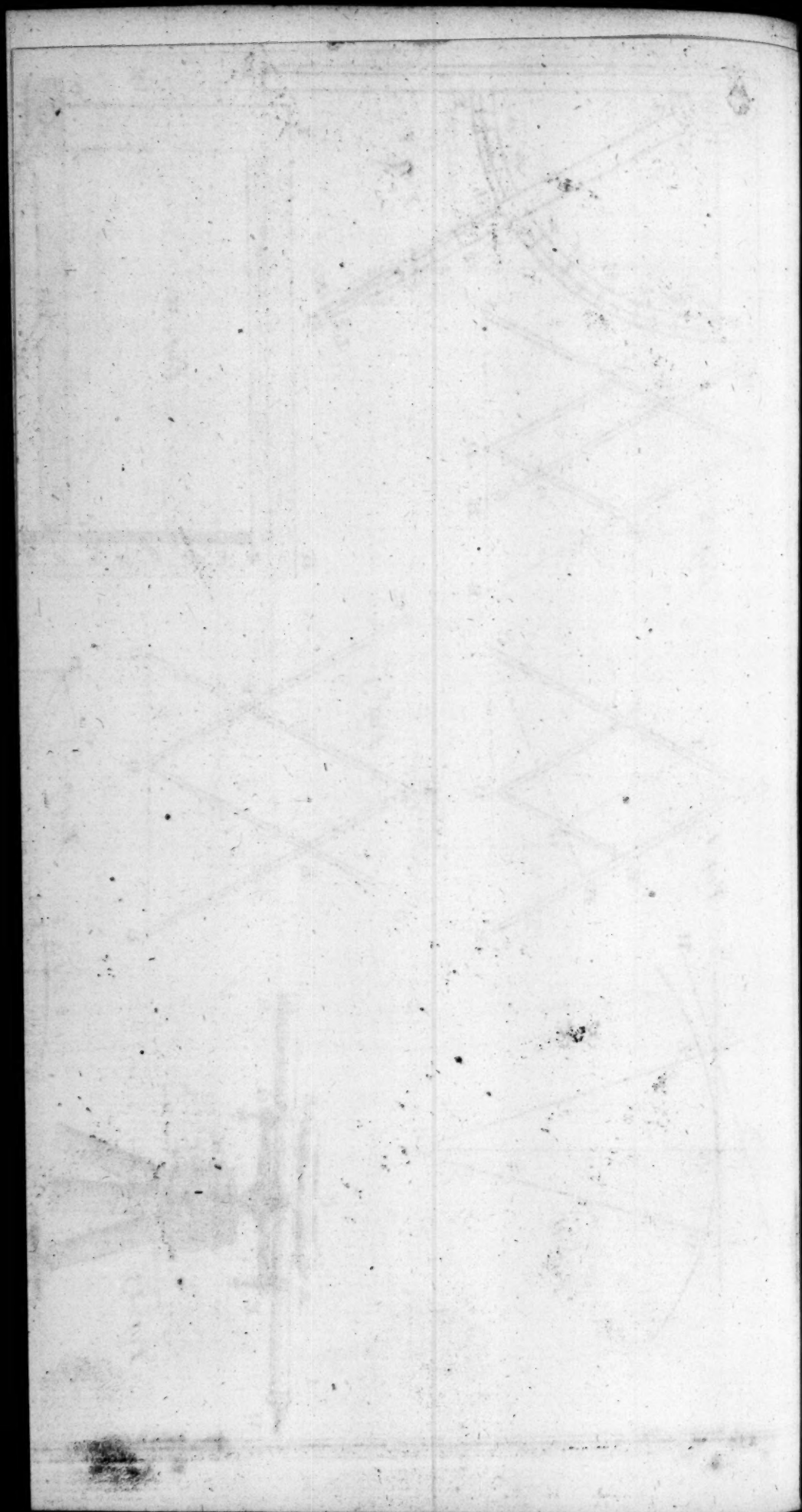


Fig. 12.



also be known as before, and this may be called the second *auxiliary Angle*; and the Difference of these auxiliary Angles, is the true Quantity of the Angle $P O Q$ to be measured. But this is as yet but a *single Measurement* though different from, and much more exact than by the common Method.

But to measure this Angle more accurately by taking its *Multiple*, let *Fig. 2.* represent the above Case of the Angle *once taken*; that is, let $b O w$ be the first auxiliary Angle, $b O r$ the second; and the Angle to be measured, their Difference $w O$, or $P O Q$. Then loosening the Screw below, the two Rulers become easily moveable, and (retaining the same opening) they are to be so posited now, that the Telescope may be directed to the first Object P , and the Whole then fastened by turning the Screw below. In this Position of the *Goniometer*, let the Telescope be again directed to the Object Q , and in this Case the Angle will be *twice measured*, that is, the Arch $w 2$ is equal to twice $w 1$, or $w O Q = 2 P O w$, as shewn in *Fig. 3.*

In this Situation of the Instrument let the Screw below be again loosened, and the Whole turned about 'till the Telescope comes again to the Object P , and let it be there made fast; then the Ruler and Telescope being carried round to the Object Q , let it there rest; and the Angle is now *tripled* as shewn by the Arches $w 1, 12, 23$, in *Fig. 4.*

After the same Manner you proceed, and *quadruple* it, as represented in *Fig. 5.* and *quintuple* it, as in *Fig. 6.* And, lastly, you *sextuple* it as in *Fig. 7.* But in all this Time, it is plain, the

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the Ruler $c d$ from its first Position, *Fig. 2.* to the last *Fig. 7.* has passed through the Circumference of a Circle, and the overplus Arch $w 6$.

Now this last, or second auxiliary Arch $b 6$, being measured upon the Scale becomes known, from which therefore take the first $b w$, and the Remainder $w 6$ added to 360 Degrees, will be equal to six times the true Quantity of the Angle $P O Q$, which will therefore be known.

For Example, let the first auxiliary Angle $b w$ be ten Degrees (for this may be taken at Pleasure) and suppose the last auxiliary Angle $b 6$ be thirty-four Degrees thirty-five Minutes; the Difference between these two is the Arch $w 6$ = twenty-four Degrees thirty-five Minutes, which added to 360 Degrees, the Sum is 384 Degrees 35 Minutes, the sixth Part of which is 64 Degrees 5 Minutes 50 Seconds, the Quantity of the Angle $P O Q$, as near as *the Mean of a sixfold Multiple* will give it.

When the Angle is not large, as from 20 to 40 Degrees, it will be sufficient to take the auxiliary Angle that over-runs the Semi-circle, instead of the whole Circle: for in this Case, the Labour would be too great, and the Accuracy superfluous, since an Angle of 32 Degrees six times repeated, is but 12 Degrees over a Semi-circle, and an Angle of 20 Degrees, multiplied ten times, is but 20 Degrees more than the Half-circle; and an Angle of 16 Degrees may be repeated twelve times within the Semi-circle and 12 Degrees over. Further, if the Angle be yet smaller, as 10 Degrees, you may use the Quadrant only, and the auxiliary Angle beyond it. And on the other Hand if the Angle be very large, as 80 Degrees, it may be

be still with Ease repeated in two or three Circumferences of a Circle ; or one and a Half, two and a Half ; or one Quarter, or three Quarters, at the Pleasure of the Artist.

The Instrument as hitherto considered, will be contained in a Box thirteen Inches long, two and a Half Inches wide, and about the same Depth ; is by far more compendious and portable than any other Instrument yet used in surveying even of the most trivial Sort ; and, in Point of Accuracy, far exceeding the so much famed *Theodolite* of Sissons, which has always sold for twenty Guineas, though but ten Inches Diameter, and a *Nonius* giving an Angle (by single Measurement) only to three Minutes of a Degree.

I might here also observe that this *Goniometer* may be constructed with a *Nonius* instead of a *Scale of Diagonal Lines* ; and I apprehend great Advantage would accrue thereby ; but this is to be obtained in a Form very different from those of our common Instruments, and I cannot but wonder how Mr. MAYER could treat this Subject without the least Notice of the *Nonius*. There are some *Punctillio's* relative to the perfect Construction of the *Goniometer*, which he is also wholly silent about, and unless they are carefully observed by the Maker of these Instruments, the Surveyor will have much Trouble and little Certainty in the Use of it.

Such is the Nature of the Instrument or *Goniometer* here recommended and described from the learned MAYER : But that the Reader may have the Sense and Judgment of the whole ROYAL SOCIETY of GOTTINGEN upon it, I shall

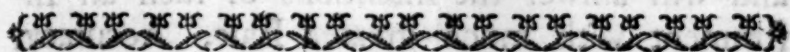
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shall conclude this Tract with transcribing their Sentiments from the *Latin* Preface to the second Volume of their COMMENTARIES, which is thus,

“ All who have been hitherto concerned in perfecting Instruments for measuring Angles seem to have sought all the Advantage from their Bulk or Magnitude alone : Hence the very great Difference between *geometrical* and *astronomical* Instruments. For the latter, since it is necessary they should be fixed and continue in one and the same Place, may be constructed of as large a Size as we please ; so that it is no Wonder if they are carried to the highest Degree of Perfection, and are adapted to measure Angles to a very few Seconds.

“ But the former (*viz.* geometrical Instruments) being necessarily made small, in order to be portable, are for that Reason much inferior to them in point of Accuracy. Neither was it possible to be vulgarly credited that the same Degree of Perfection could be given to an Instrument of a few Inches Radius, which we obtain not without many Artifices, from those of a Radius many Feet in Length. It is therefore a singular Compendium, which will not a little conduce to a safe geometrical Praxis, which the excellent Mr. MAYER has introduced into these small Instruments. It is neither sumptuous, nor requires any thing more than what commonly enters their Structure, except one small Telescope only. And yet nevertheless so great is its Efficacy, that any terrestrial Angle (*viz.* one whose containing Lines are fixed) may be thereby measured without an Error of more than 10" or 15". And that he might illustrate this

this Method by an Example, he has explained the most easy and simple Structure of the Instrument with the Proceſs of uſing it by Copper Figures."



CHAP. IV.

The beſt CONSTRUCTION of Instruments required in PLOTTING or making PLANS of a SURVEY, with the THEORY on which they depend.

W HEN the Eſtate is ſurvey'd, a fair PLAN or *Draught* of the Whole is to be made by plotting, or taking off from a proper *Scale of equal Parts*, the Meaſures of all the *Right LINES*; and from another Instrument the various *ANGLES* muſt be *protracted* or meaſured off; and this Instrument is therefore called the *PROTRACTOR*; as the other is called the *plain SCALE*.

Besides theſe, it will be neceſſary to have at hand two or three other Instruments, *viz.* a *Square*, a *Parallel Ruler*, a *Pantagraph*, or a *Pair of Proportional Compaſſes*. But there will be no Occaſion to dwell on a Deſcrip-

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tion of the *plain Scale* or the common *Square*, as being sufficiently known to every one; and also the *parallel Ruler*, of the usual Form, is in every one's Hands: but that which I would recommend to the ingenious SURVEYOR, is of a different and far more commodious Construction, and will answer the Intentions of such an Instrument with much greater Ease, Accuracy, and Expedition.

The Form likewise of the common *Protractor*, as being only a Semi-circle or whole Circle in Brass, divided into Degrees, has nothing in it difficult, regarding either its Use or Construction. But that Instrument which I here propose is one in which all the three, *viz.* the *plain Scale*, the *Square*, and the *Protractor* are contained, or combined together.

There remains therefore only the PANTAGRAPH to be described and constructed from a Mathematical Theory, which is as follows; let A B and A C denote two thin Slips or Bars of Brass, Wood, &c. freely moveable upon a Joint at A; and let two other Slips D E, and D F, of half the Length of A B, be so connected at D, E, and F, as to be moveable with the utmost Freedom on those Points. By this Means the Part A E D F will be a *Rhombus* with Angles accute and obtruse in every Degree, by opening or closing the Bars or Legs A B and A C, in the Use of the Instrument.

Through a Point G, taken at Pleasure in the Bar D E, draw the dotted Line C G H; then it is plain, that if the Point H be made the *Center of angular Motion* to the whole Instrument or PANTAGRAPH B A C, the Points C, G, H, in all Openings of the Bars A C, A B will be
in

in a *Right Line*; and therefore when the Instrument moves about the Center H, the Velocity of the Points G and C will be as the Distance GH and CH; and further that the Motions, or Lines described by the Points G and C in the Movement of the Instrument, will be always as those Distances from the Center, and of Course, will be similar to each other.

The Reason of all which follows hence, *viz.* that drawing the Line GI parallel to AB, and ED being parallel to AC, we have the Triangles ACH, ICG and EGH constantly similar to each other; and therefore these Points G and C must ever move in a similar Manner: and $HG : HC :: AI : (= EG) : AC$, but in all Openings and Positions of the Instrument about the Center H, the Ratio of EG to AC is the same; therefore so is the Ratio of HG to HC.

Since H is made the Center of Motion, if a Plan or Drawing be laid under the End of the *Pantagraph* at C, and a Sheet of clean Paper at G, with a Crayon over it; a Tracer at C being moved over all the Contours and Lineaments of the Plan, will cause the Crayon or Pencil at G to describe a similar Figure on the Paper, and the *Linear Dimensions* of the Copy will be to those of the *Original* or *Plan*, as the Line GH to the Line HC; or it will be diminished in the Ratio of GH to HC.

On the contrary, if the Plan be placed at G, and the Blank Paper at C with the Crayon, then the Tracer at G moves the Crayon at C over the Lines of a Copy enlarged in Proportion of HC to HG.

Again, if G be made the Center of Motion, it is plain that the Tracer and Crayon being placed alternately in C and H, will produce a Copy from the Original that shall have thereto the Ratio GH to GC, or of GC to GH, as you please.

It is farther evident, that when the Point G coincides with D, the Line CH becomes CB; and if D be the Center of Motion, the Copy at C will be equal to the Plan at D, because $CD = DB$. But if B be the Center of Motion, then the Copy at D is but half so big as the Plan at C, and *vice versa*.

In this Construction of the *Pantagraph*, the Point H must be determined corresponding to the Points C and G, which will divide the Line EB as it ought to be, for fixing the Center at H or G for any proposed Ratio of the Plan and its Copy. This is done by the following Analogy, *viz.* $IC : IG :: EG : EH$, that is $IC : IA :: AE : EH$. But the Ratio of IC to IA is that of GC to GH, *viz.* that of the Plan to the Copy, or Copy to the Plan, and
$$\frac{IA \times AE}{IC} = EH$$
 for that assigned Ratio; and thus it is found for all others.

But this Construction of the PANTAGRAPH is not so simple and elegant, as the following (*Fig. 2.*) where AEDE is the same *Rhombus* (as before) upon the Side AC; and HK is a Bar equal to AC, and moveable upon the Sides of the *Rhombus* ED and FD *parallel-wise*; and where it may be occasionally fixed in the Points G and I.

In this Form we have the Triangles CFD, DGH alway isosceles and similar; and therefore

fore $CF : CD :: DG (= FI) : DH$, or $HC : HD :: IC : GD = IF$. Hence if D be the Center of Motion, and the Plan and Tracer be placed at C, the Crayon at H will delineate a Copy thereof diminished in Length and Breadth in the Ratio of HD to HC; and in that Ratio it may also be encreased if the Plan be laid under H, and the Crayon delineates at C.

In this Construction the Divisions are all equal on every Part, and therefore it admits of great Ease and Uniformity, and even the Exactness of a *Nonius* if required; so that the Lineal Proportion of a *Plan* and its *Copy* may be varied from 1 to 100, and that to the 100th Part of an Inch.

But this PANTAGRAPH is yet not so simple as the Construction of (*Fig. 3.*) where AC, AB, and DG, DH are two Pair of Bars, connected and moveable about the Points A and D, and placed upon each other in an inverted Position, capable of being fixed in, and moveable upon the Points E and F, so that AEDF may always be a *true* PARALLELOGRAM; then if C be the Center of *Angular Motion*, and the *Plan* and *Tracer* placed at B, the Copy at D will be drawn and diminished in the Proportion of CD to CB, or it may be as much enlarged, if it be placed at B and the Plan at D.

In this Case, too, CFD, CAB are similar and isosceles Triangles, which give CA to CF as CB to CD, or as the Drawing at B to that at D. Whence in all the *three premised* CONSTRUCTIONS of a PANTAGRAPH 'tis evident that if AB be divided into 100 equal Parts, and the other Legs and Parts in the same Manner, then may the Copy be drawn in
any

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any Proportion to the Plan from 1 to 100, as was said before,

For Example, it is required to diminish the Drawing of a Plan in the Ratio of 2 to 5, or of 4 to 10, that is, of 40 to a 100, then AC being 100, you make $FC = FD = AE = 40$; and then we have $CB : CD :: 100 : 40 ::$ Plan at B to the Copy at D, as required. But in the PANTAGRAPH of Fig. 2. it is $CF : DG :: 50 : 20 :: DC : DA ::$ Plan at C : Copy at H.

If it be required to diminish or encrease the *Area* of the Plan in any given Ratio, then a Line expressing the *Square Roots* of Numbers must be placed upon the Leg AC, &c. because the Sides of similar plain Figures are as the Square Roots of their Areas respectively. Thus for Example, suppose it required to *diminish the Area of the Plan to one fourth Part*, or in the Ratio of 1 to 4, then the Side of the Copy will be to a like Side in the Plan, as $\sqrt{1}$ to the $\sqrt{4}$ or as 1 to 2; therefore $CF = \frac{1}{2} CA$ in Fig. 3. or $DG = \frac{1}{2} CF$, or ED, in Fig. 2. Again, let the Area of the Copy be to that of the Plan as 1 to 9, then the like Sides in each we be as 1 to 3, and therefore $CF = \frac{1}{3} CA$; and so for any other Ratio. Whence the Reason of the *Line for Areas* on some of those Instruments is evident; as also the Method of proceeding to do the same Thing *without such a Line*.

The next Instrument to be described, and whose Structure is to be explained from a *Mathematical Theory*, is that of Fig. 4, which (as was said) contains three several Instruments, viz. 1. The common SQUARE ABD, which requires

requires no Explication. 2. A PLAIN SCALE on the Leg or Part B D; on this Scale a Variety of useful Lines are placed, such as are required in the Practical Mathematical Arts, and those which relate to *Plotting*, are supposed to be understood by the SURVEYOR. 3. A PROTRACTOR, by Means of the Arch F G, and Index I M.

The Nature of this *Protractor* requires an Elucidation from THEORY, in that Part which concerns the Graduation of the Index into *Minutes*, more especially as no author (that I know of) has treated this Subject in such Manner as is necessary for duely understanding it. The two Sides of the Index are parallel from the center C to the Limb of the Quadrant E G, where the Side I H instead of going on to N in a right Line, is carried towards M, either in the Arch of a Circle, or in a strait Line H M; but in either Case the Arch M N is *just one Degree* of a Circle whose Radius is C N.

If H M be a *Curve* it must be divided into 60 *equal Parts* for the *Minutes* of a Degree; but if the said Line H M be a *strait Line*, the Division will be into 60 unequal Parts. All which will be evident from the following Constructions. Upon the Center C (*Fig. 5.*) describe the Arch N M, which suppose is required to be divided into 6 equal Parts; in the Radius C N take any where the Point H, and then through the three given Points C, H, M, draw the Arch of a Circle C H M, and divide the Part H M into 6 equal Portions at the Points 1, 2, 3, 4, 5, 6; and through these Points draw right Lines from the Center C, and they will divide the Arch N M into 6 equal Parts as required,

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required, for equal Angles at any Point C in the Periphery of a Circle will be subtended by, or stand upon equal Arches H 1, 12, 23, &c. in any other Part of the same Periphery; by Prop. 27. Book III. of *Euclid*.

But if the Line H M be a strait Line, it will be the *Chord of the Circular Arch* H M; and then the Radials C 1, C 2, C 3, &c. which divide the Angle N C M into 6 equal Parts, will divide the *Chord Line* H M into 6 very unequal Parts, in the Points, 1, 2, 3, &c. Each of which may be computed by plain Trigonometry for an Angle N C M = 60 Minutes, or 1 Degree. Thus let C M = 8 Inches, and C H = 5; then in the Oblique Triangle C H M, there are given two Sides and an Angle included, whence the Angle H M C will be found to be 1 Degree 40 Minutes; and the Angle N H M = 2 Degrees 40 Minutes. Now make the Angle H C, = 5 Minutes; then in the Triangle H C 1, there are known all the Angles, and the Side C H, by which Data we find the Side H 1 = 0,1613 of an Inch, answering to the first 5 Minutes. And thus all the Divisions are found for every five Minutes along the Line H M.

Now by this Protractor it be easy to lay off an Angle to a Minute; for Example, at the Point C in the Line C E, let it be required to make an Angle of 28 Degrees 35 Minutes; place the Center of the Protractor upon C, the internal or fiducial Edge of the Leg C D upon the Line C E, then holding it fast, move the Index upon the Limb 'till the Side I H coincides with the 28th Degree, and there let it rest; then with the Point of the Compasses make
a fine

a fine Point at 35 Minutes in the H M, through which, and the Point C, draw a Line and it will make the Angle required with the Line C E.

As there is no *Mathematical Praxis* in which the Use of a PARALLEL-RULER is more frequent and necessary than in *Plotting, reducing, and measuring* SURVEYS, therefore I shall here describe one of a new Construction, with a *Drawing* TABLE adapted to it, both which together are represented in *Fig. 6*. The Ruler is denoted by A B C D E F, and the Table by G H I K.

This *parallel Ruler* is a *double one*, consisting of the two single Ones A B E F, and E F C D connected together; by this Means it will admit of being opened to a *great Extent*, even that of the whole Table if required; and with this Convenience, that the moveable Part C D may be carried perpendicularly over the fixed Part A B, or obliquely on one Side or the other, as Occasion requires, to a Distance twice as great as a *common Ruler* of the same length can reach.

As the Side of the Ruler A B may be fixed by two *steady Pins* upon the Side of the Table G K, while the other Bar C D moves with its End A contiguous to the Side G H of the Table; therefore it is plain, that if a Scale of Inches and Tenths O P be fixed upon the said Part of the Table G H, and a *Nonius* upon the End A sliding adequately by it, then parallel Lines may be drawn at any given Distance to the Exactness of the $\frac{1}{1000}$ Part of an Inch.



C H A P. V.

The Method of reducing any Multangular PLAN or SURVEY to one Plain TRIANGLE; and finding its whole CONTENT by one COMPUTATION.


 AFTER a Survey is plotted, in order to measure the same, it has been customary to reduce it into several *Trapeziums* and *Triangles* for that Purpose; but as a Multitude of Measurements of *Bases* and *Perpendiculars* cannot be made without manifold Errors (in Parts so small as *Links* must be) it will be the best and surest Way by far to reduce the Whole irregular Polygon (or Plot) to *one TRIANGLE only*; which is done in the following Manner by the *Parallel Ruler*.

To reduce any Quadrilateral (or four-sided) Figure (7) A B D C to a Triangle. Continue out the Side B A towards E; and draw A D, and parallel to it C E; join D E, and the Triangle B D E is that required. For the Triangles A D C, A D E, upon the same Base A D, and between the same Parallels A D, C E, are equal

equal (by Prop. 1. Book VI. of *Euclid*;) and by adding to each the common Triangle ABD , we have the Triangle BDE equal to the quadrilateral $ABCD$.

Again, to reduce an irregular Pentagon AD (Fig. 8.) to a Triangle upon a given Side AB . Continue the Base AE towards G ; and draw the Line CE , its parallel DE , and the Line CF ; substitute the Triangle CEF for its equal CDE ; and the quadrilateral $ABCF$ will be equal to the Pentagon. Then draw BF , its Parallel CG , and the Line BG ; and putting the Triangle BFG for its equal BFC , you have the Triangle ABG equal to the Quadrilateral $ABCF$, and consequently to the Pentagon $ABCDE$.

Lastly; Suppose the Plot a Heptagon, or seven-sided Figure (like Fig. 9.) then continue out the Sides AB , BC , CD , and DE ; draw GE and its Parallel FH ; GD and its Parallel HI ; GC and its Parallel IK ; GB and its Parallel KL ; and drawing GL , it will make the Triangle AGL equal to the Heptagon $ABCDEFG$.

To demonstrate the Truth of this, first draw GH , and then is the Triangle $GEH = GEF$; add the common Hexagon $ABCDEG$, and it makes the Hexagon $ABCDHG$ equal to the Heptagon $ABCDEFG$.

Secondly, draw GL ; then is the Triangle $DIG = DHG$; and by adding the common Pentagon $ABCDG$, you have the Pentagon $ABCIG$ equal to the Hexagon $ABCDHG$.

Thirdly; draw GK , then is the Triangle $GKC = CIG$; add the Quadrilateral $ABCG$, and you have the Quadrilateral $ABKG$ equal to the Pentagon $ABCIG$.

E 2

Fouthly;

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Fourthly; the Triangle $BLG = BKG$; add the common Triangle ABG , and you have ALG equal to the Quadrilateral $ABKG$, and consequently to the *Heptagon* proposed. Q. E. D.

By these Examples, the Surveyor will readily observe the practical Method of reducing his *polygonal* PLOT, or PLAN of an ESTATE, MANOR, COMMON, &c. to one simple TRIANGLE; and at the same Time how necessary the Use of the before-described *Double PARALLEL RULER*, in all such Operations, must be.

As in *Surveying Land*, the Measures are taken in *Chains* and *Links*, and the Contents computed in Square Links, and then reduced to *Acres*, *Roods*, *Poles*, or *Perches*, square Measure; it will be proper here to observe, that an *Acre* consists of 160 square Poles; and that as a Pole contains $16\frac{1}{2}$ Feet, 4 Poles will contain 66 Feet, and this is the Length of the CHAIN in which the Measures are taken; this Chain is divided into 100 Links; and consequently each is (or ought to be) $\frac{66}{100}$ of a Foot, or 7,92 Inches. Hence a square Pole contains 625 square Links; and an Acre contains $625 \times 160 = 100000$ square Links.

And hence appears the Reason why you alway cut off 5 Places for Decimals, leaving the Number on the left Hand for Acres in the Content of any Plot of Ground reduced to square Links. Thus suppose you have in the Content of a Field 1235745 square Links, this must of Course make 12,35745 Acres, that is, (by reducing the *Decimal Parts*,) 12 Acres, 1 Rood, 17,2 Poles. Any Thing farther upon this Head will be unnecessary even to a *Tyro* in Surveying.

As

As the Forms of Hedges and Sides of Fields and Inclosures seldom consist of long strait Lines, but either short ones, or such as have some Degree of *Curvature*; if the curved Boundary be of an irregular Form, it may be reduced to a Right-lined one, by drawing Chord-lines nearly to coincide with the Curve in its several Parts, and the Deficiency or Overplus in the Computation of the Contents of such Segments of Curves will be very inconsiderable by a judicious Management.

If the Boundary be rectilinear, consisting of many short Sides, as BC, CD, DE, EF, &c. (Fig. 10.) then by drawing the strait Line BF, you may reduce the whole Space between that Line and the irregular Boundary BCDEF into one Triangle by the Method above taught, which I shall in this Case also illustrate.

Lay your *Parallel-Ruler* from B to the second Angle D, and then raise the moveable Edge to the Angle C, and where it cuts the Slide AB (continued out) make a Mark, as at (*b*) and draw D*b* crossing BC in *d*. Then it is evident the Triangle B*d**b* is equal to *d*DC, and therefore the Figure B*b*DEF of five Sides, is equal to the first of Six.

Again; lay the Edge of the Ruler upon *b* and E, and holding it fast, raise the Edge to the Angle D and make a Mark at (*a*) where it crosses the Line *a*B produced; and draw the Line *a*E intersecting D*b* in (*e*); then the Triangle *a**e**b* = *e*DE; and now the Figure of 4 Sides B*a*EF is equal to the foregoing one of five.

Lastly; lay the Ruler over the Points, *a*, F, and move the Edge to the Angle E, and it will
cut

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cut the Line A B in (c) ; draw c F cutting a F in (b) ; then the Triangle $a f c = f E F$; and consequently the Triangle B F c is equal to the Quadrilateral B a E F, and therefore to the irregular Area bounded by the right Line B F and the *Zig-zag* Side BCDEF.

S C H O L I U M.

The Use of the GONIOMETER has been hitherto considered only in regard to SURVEYING, where *horizontal Angles* are to be measured; But the Instrument may be constructed to take Angles in a *vertical or any Oblique Position*; and so may be used for all the Purpose of ALTIMETRY; as also in many Problems of ASTRONOMY, particularly that famous one of taking the Distance of the MOON from the SUN or STAR, upon which the PROBLEM of *finding the LONGITUDE* so much depends. And because this Distance is required with the greatest Accuracy possible, 'tis evident the GONIOMETER is better by much than even HADLEY'S OCTANT for such a Purpose, because by the latter you cannot attain the Measure of that Distance by *multiplying the same*, as in the *Goniometer* you may very easily; and moreover the conciseness and Simplicity of its Structure recommends it in all such difficult Processes before any Instruments whatsoever.



C H A P. VI.

*The PRINCIPLES of LEVELLING in
THEORY and PRACTICE, concisely
explained and illustrated.*

 HE Difference between the ART of
LEVELLING in general, and that of
Hydraulic LEVELLING, is, that by
the former you find a *True* LEVEL;
and by the latter you find, by Means of that
true Level, a proper CHANNEL for a WATER
COURSE. The one results naturally from the
other.

In this Art there are *two Levels* to be considered, the *apparent* and the *true Level*. To give an Idea of these, let C be the Center of the Earth (*Fig. 11.*), and AB an Arch of a great Circle upon its Surface; let the indefinite right Line KI touch the Earth in the Point A, and through B draw CG intersecting the Tangent AK in G. Then because the Point G is farther from the Center C than the Point A, a Particle of Water, or any other Body, laid at G, and affected by the Power of Gravity only, will descend from G to A upon an inclined
Plane

Plane G A, and at the Point A it must become quiescent.

Now suppose a long strait Glass Tube filled with *Water*, *Spirit*, &c. all but a small Part possessed with Air, and moveable in its middle Part upon the Point A. Then if its End towards G were raised above the Line L G, the other End must descend below it, and the *Spirit* as being the *Heavier Fluid*, will fill all the lower Part of such an inclined Tube, and the Air must rise wholly to the upper End towards G; and the same Thing will happen if the Tube were inclined the contrary Way; wherefore when the Tube is not inclined at all, but placed exactly in the Line L G, then the Air will be equally sustained by the Water at each End, in the Form of a Bubble in the Middle of the Tube over the Point A.

Such a Tube, therefore, placed upon a Telescope, and so adjusted that the Axis of one be accurately parallel to the Axis of the other, and then both together placed upon two parallel Plates, which by *adjusting Screws* may easily be so disposed, that the *Bubble of Air* in the Tube shall be made to rest in the Middle Part; and then it is plain the Axis of the Telescope will be in the Line L G; and therefore an Eye will view through that Telescope any Object-place in that Line at G, K, &c. which Line is therefore called the *apparent* or *horizontal LEVEL*.

But a *true* or *real LEVEL* must be that which is every where equally distant from the Center of the Earth; and therefore must be an Arch of a Circle A B upon the Surface, or parallel to t. Because upon such a Level or Curve, a
Body

Body can have *no tendency to Motion*, being everywhere *equally affected by the Power of Gravity*.

From what we have said, 'tis evident, that the Difference between CA and CG , which is BG , is the *Perpendicular Height* of any Object G above the *true Level* of the Surface of the Earth, or rather of the SEA, at the Point A . If the Length of the Arch AB be known, then in the right angled Triangle GAC , the Base $AC (= BC)$ being *Radius*, AG *Tangent*, and CG *Secant* of the given Angle ACG ; the Height BG is the Difference between the *Radius* and *Secant*, and is always known by the *Trigonometrical Canon*.

If G be a SPRING, or FOUNTAIN-HEAD, then by the *Spirit Level*, you find the Position of the *Horizon* or *apparent Level* EH , passing through the Point G ; and by moving the Telescope 'till through it you see the Point A on the *Sea-Shore* you will find the Quantity of the Angle EGA by the *Goniometer*, which will ever be equal to the Angle ACG , and thus the Height of the Spring or Fountain at G above the Surface at B , is readily known; and of Course the Distances AB , and AG .

By Calculation it will appear, that when the Arch AB is equal to 5280 Feet, or *one Mile*, then the Angle $ACB = 0^\circ : 0' : 52''$, and the Height $BG = 7,409$ Inches; the Water in a Canal or River of $7\frac{1}{2}$ Inches Descent in a Mile will move but slowly; to give the Water a proper Velocity, it has been found by Experience that 12 or 15 Inches per Mile below the Horizontal Level, is necessary for Rivers in general.

F

If

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If A be supposed upon the Sea-coast, it is plain, no Water from the Sea can be carried in any Canals but what are below the *true Level* A B of the Earth's Surface; nor can any Streams or Currents of Water flow but from a Spring at G above that *true Level*; and lastly, that if A B be an Arch of 20 Miles, the Height of the Fountain-Head B G must be at least 20 or 25 Feet.

As to the practical Method of Levelling, there is but little required to be said. For if a *Station-Staff* of 10 Feet Length be divided into 1000 equal Parts, each Foot will contain 100; and each of these Parts will be larger than the tenth Part of an Inch, for of these there are 120 in a Foot.

If therefore it be required to know how much the Hill A is higher than the Hill B, (*Fig. 12*); a *Station-Staff* G H is placed upon the former, and another F I upon the latter; and then the SPIRIT-LEVEL D is to be placed upon some Eminence at K, where the Vanes E and C may be plainly seen from thence. In this Operation there are two Assistants, one at each Staff; otherwise the Work will be very tedious. When you have directed your Level to the Staff at A, and adjusted it properly, you give Orders by a Signal for the Assistant at A, to slide the Vane E up and down till you see the Intersection of the Lines upon the Vane coincide with the Intersection of the Hairs in the Telescope, which the Assistant is apprized of by another Signal, and the Vane remains at the Height G E upon the Staff.

Then the Level is turned about to the other Staff upon the Hill B; and the same Operation being

being there repeated, the Vane rests in the Position at C. Then is the Line C D F, passing along the Axis of the Telescope, the *apparent Level*; and the higher Ground being A, draw from the Foot of the Staff G the Line G O parallel to the Line C E; and then it is plain F O will be the Difference of the Heights of the two Hills A and B, and will shew how many Feet, and hundredths of a Foot, the Place F is lower than G.

If the Descent from G to F be found to give too great a Velocity to Water in a right lined Canal, the Velocity, or Force of Water, may be greatly diminished by carrying it in Pipes down the Hill A, across the Vale L, then laid up and carried over the rising Ground, and through the Vale N to the Reservoir at E.

In order to find a *true Level*, regard must be had to the *Curvature of the Earth*, and since Distances are measured by the Chain of 4 Poles, or 66 Feet, you will see in the following Table the Quantity of Curvature in Inches below the *apparent Level*, at the End of any Number of Chains to 100.

N. B. The Numbers of this Table are calculated upon the well known Principle, *That the Subtenses B G, D L of the tangential Angle are as the Squares of their respective Arches A B and A D*, that is, $B G : D L :: \overline{A B}^2 : \overline{A D}^2$.

TABLE

The New ART of SURVEYING

TABLE of CURVATURE.

Chain	Inches	Chain	Inches	Chain	Inches	Chain	Inches
1	0,00125	14	0,24	27	0,91	40	2,00
2	0,005	15	0,28	28	0,98	45	2,28
3	0,01125	16	0,32	29	1,05	50	3,12
4	0,02	17	0,36	30	1,12	55	3,78
5	0,03	18	0,40	31	1,19	60	4,50
6	0,04	19	0,45	32	1,27	65	5,31
7	0,06	20	0,50	33	1,35	70	6,12
8	0,08	21	0,55	34	1,44	75	7,03
9	0,10	22	0,60	35	1,53	80	8,00
10	0,12	23	0,67	36	1,62	85	9,03
11	0,15	24	0,72	37	1,71	90	10,12
12	0,18	25	0,78	38	1,80	95	11,28
13	0,21	26	0,84	39	1,91	100	12,50

Hence it appears, that if the Distance GO be 20 Chains, we have $FO = 0,5 = \frac{1}{2}$ an Inch; but if $GO = 40$ Chains, then $FO = 2$ Inches; that is, a Line drawn from G to F will in that Case be a *true Level*. Therefore a Canal cut from G towards F must at the Distance GF be more than 2 Inches below the horizontal Level GO for the Water to descend, or run therein, from a SPRING at G . If $GF = 100$ Chains then $FO = 12\frac{1}{2}$ Inches, consequently at the End of 100 Chains, or 6600 Feet, there must be a Descent of more than a Foot below the *apparent Level*, as was before observed.

As I think what has been said contains the THEORY of LEVELLING in all its essential Parts, I shall conclude with only observing that *Fig. 13. (Plate II.)* represents the best Form of a *telescopic Spirit Level* in common Use; where AB is the LEVEL, with it's Bubble adjusted to the Middle Point C . The TELESCOPE is denoted by DE ; and FG the Part to which it is fixed on the three Legs; and lastly, H, I , are the two adjusting Plates, for placing the Instrument *horizontal* by the Screws which appear in them.

F I N I S.